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CONTENTS

G. E. Moore: Some Impressions
AUSTIN DUNCAN-JONES

The Rules of Natural Deduction
J. L. MACKIE

Sea-Fights Without Tears
JOHN KING-FARLOW

Imperatives and Deontic Logic
HECTOR NERI CASTANEDA

THREE SHILLINGS AND SIXPENCE NET

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G. E. MOORE: SOME IMPRESSIONS

By AUSTIN DUNCAN-JONES

THE remarks which follow are meant only to supplement the formal memoirs which have appeared and will doubtless appear. I began to go to Moore's lectures at Cambridge in 1930. At his first lecture I somehow divined that note-taking was out of place: until then, being callow and docile, I had always taken notes slavishly. The result was that even quite a short time afterwards I should have been hard put to it to say what Moore had lectured about. The title of the course was ' Metaphysics ', and the Theory of Descriptions must have come in a good deal. The phrase 'ordinary language' has long since become tedious. But in those days it was exhilarating to hear Moore quote from Russell: " ' Whom did you meet? ' ' I met a man.' ' That is a very indefinite description.' "; and continue " *of course* it's not an indefinite description; it isn't a description *at all.*" Moore's style in lecturing has been well described by Susan Stebbing in *The Philosophy of G. E. Moore*. He wrote sets of sentences on the blackboard, talking about the distinctions between them as he wrote. Among all his writings I think " External and Internal Relations " most suggests what his lectures were like. He struck me as a shy person; and shyest, I fancied, when he lectured on a Scarlet Day and wore a doctor's full dress gown. He was not difficult to talk to in his conversation classes. After a time Moore asked me to tea. It was different from the Cambridge tea parties I was used to, and more like a conversation class: I don't think there was any of the local gossip or bookish chit-chat which I'd enjoyed elsewhere. For many years a notice in the Cambridge lecture list said that Moore was at home to members of the University at 5 on Tuesdays. In my time no one ventured to go. Some years later a new generation of Moral Scientists was bolder, and Moore's Tuesdays became a flourishing institution.

The Moral Science Club in those days was the scene of the tragi-comedy of Moore and Wittgenstein. When Wittgenstein was there he monopolised the discussion, and most other people, Moore among them, were tongue-tied. This was vexing both for Wittgenstein's critics and for Moore's admirers. By the expedient of starred meetings, from which senior members were debarred, Wittgenstein was sometimes kept out: but that was no

good to the party which wanted to hear Moore. There were times, however, when Wittgenstein stayed away and Moore's characteristic chairmanship was displayed. After a paper had been read Moore would sit silent for a few minutes, frowning with perplexity. Then he would mention something in the paper which he had not understood. He often started quietly and even haltingly, and grew more rapid and vehement as the enormity of the speaker's muddles began to take possession of him. If there had been anything forensic or debating in it, Moore's combative style in discussion might have been unpleasing. But Moore's force of manner came from his passion for clarity. Like Socrates he often declared his own ineptitude and confusion: but unlike Socrates he never tripped up a disputant.

Moore spoke often and vigorously at Joint Sessions. To young Moral Scientists in the early thirties Joint Sessions seemed like ceremonies presided over by the pontiffs of an outlandish cult. While most of the auditors applauded, a knot of malcontents was silent in a corner. They were waiting for Moore, the only champion of light: and when he rose to speak the applause had quite another note.

As Moore grew older I saw him less often, so what I record is mainly the impression made by an elderly man on a very young one. To young people the old and distinguished often seem like figures on a stage, and if they seem so ordinary human contact is hindered. I think it was from Moore that I first learnt the intellectual excitement which belongs to conversing with someone whose powers are far beyond one's own. Such things are hard to convey.

A memoir on Moore by Professor J. Wisdom will be published in the next (January) number of ANALYSIS.

THE EDITOR.

THE RULES OF NATURAL DEDUCTION

By J. L. MACKIE

THE set of procedures known as "Natural Deduction" or "Natural Inference" provide a powerful method of logical demonstration. These procedures, "universal instantiation" (U.I.), "universal generalisation" (U.G.), "existential generalisation" (E.G.), and "existential instantiation" (E.I.), and the rules governing them, have been set out in various textbooks, but there are still problems about the meaning of these procedures and the justification of the rules.

I. The meaning of "universal generalisation".

Nicholas Rescher has argued[1]—in my opinion, correctly—that any talk of "arbitrarily selected individuals" is inept and misleading, that if s is said to be an "arbitrary element" it is not an "element" or individual at all, but a notational device, a surrogate for individuals. But he has not explained adequately what this notational device represents, or how the idea that lies behind the phrase "an arbitrarily selected individual", as used by logicians and mathematicians, can be both useful and logically sound.

Such a phrase is used in formulating the valid argument form called "universal generalisation": we can argue from ϕy to $(x)\ \phi x$ "where 'y' denotes *any arbitrarily selected individual*." At least three explanations of this process can be given.

(a) Universal generalisation normally occurs as a later step in an argument whose earlier steps have included one or more uses of universal instantiation. For instance, we may have the argument:

1.	$(x)\ [Ax \supset Bx]$	
2.	$(x)\ [Bx \supset Cx]$	$\therefore (x)\ [Ax \supset Cx]$
3.	$Ay \supset By$	1, U.I.
4.	$By \supset Cy$	2, U.I.
5.	$Ay \supset Cy$	3, 4, by hypothetical syllogism
6.	$(x)\ [Ax \supset Cx]$	5, U.G.

[1] 'Can there be random individuals?', ANALYSIS, 18.5 (April, 1958), 114–117.

Now we could say that what we have done here is to tuck the original universal quantification up our sleeve, to work with the propositional functions on which it operates as if they were the propositions which would be a single set of substitution instances of these functions, and then, having derived the required formula, to produce the universal quantification out of our sleeve again. It is as if we argued:

For all x, $Ax \supset Bx$, and for all x, $Bx \supset Cx$, so for all x both $Ax \supset Bx$ and $Bx \supset Cx$. Now if Ax, Bx, and Cx were propositions, $Ax \supset Bx$ and $Bx \supset Cx$ would entail $Ax \supset Cx$. So for all x, $Ax \supset Cx$.

Considered in this way, U.I. followed by U.G. is a notational device which represents the rule: " You can calculate, according to the propositional calculus, with propositional functions just as if they were the corresponding propositions ". The use of U.I. and U.G. together amounts to applying the whole propositional calculus to whatever is contained within the scope of a universal quantifier. But, as the first sentence in the above exposition shows, we are also using another principle, that from " for all x, P, and for all x, Q " we can infer " for all x, both P and Q ". This principle is implicit in the fact that we can instantiate two universal propositions to the same "arbitrary individual " as in steps 3 and 4 above.

Despite the extreme simplicity of the example given, I believe that this account would cover every case, however complicated, of the inferring of $(x)\ \phi x$ from ϕy where the y was itself introduced by U.I.

We must remember, however, that U.I. has a different use, when it applies a universally quantified formula to a genuine individual, not an " arbitrarily selected " one. That is, if a is a real individual, we may argue from $(x)\ \phi x$ to ϕa, and then either not generalise at all from this, or generalise only existentially. This commonsense universal instantiation is a quite different process from the U.I. that is a preliminary to U.G., the instantiation to an " arbitrarily selected individual ". It might be better if they had different names, and from now on I shall call the latter " U.I. (arb) " to distinguish it from simple or commonsense U.I. But as we shall see there is a connection between them.

(b) A second explanation of U.G. can be given as follows.

In the above example we could instantiate 1 and 2 with regard to some genuine individual a, and we could then prove

that $Aa \supset Ca$, *i.e.* we could prove the required result with regard to *a*. Then we could prove it with regard to some other individual *b*. And so on. In principle we could prove the required results, the various substitution instances of $Ax \supset Cx$, for each individual in the universe in turn. And having done so we could say: " Look, we have proved the substitution instance of $Ax \supset Cx$ for every individual, and therefore $(x)\ [Ax \supset Cx]$ ".

Considered in this way the ϕy from which U.G. proceeds is, as Rescher puts it, " a shorthand synopsis of a multiplicity of statements ", it stands for ϕa and ϕb and so on. So when we use U.G. it is as if we were collecting together an infinite set of results, each of which has been proved separately about a genuine individual by means of a commonsense universal instantiation. From this point of view U.I. (arb), the special sort of U.I. that precedes a U.G., is just an infinite set of commonsense U.I.'s.

But of course we can't really go through all the individuals in the universe. We cannot even, with safety, *offer* to do this, for fear that someone might take the offer up. All that we can really do is to sketch this programme, to indicate what it would be like. The whole of any proof, then, by U.I. (arb) followed by U.G., thus amounts to showing that the result could be proved (by means of commonsense U.I.) for any individual and therefore that it holds for every individual.

But how do we show, or see, that it could be proved for any individual? It is by seeing that in the proof for, say, *a* no use is made of any of the distinctive features of *a*, that all the propositions asserted about *a* in the course of the proof are based on what the premises have said about all individuals.

This explanation goes further than the first, because it provides a justification for what in the first was a rule arbitrarily laid down. We can now see *why* we are entitled to tuck a quantifier up our sleeve and bring it out again when it suits us. To calculate within the scope of a universal quantifier is to show how one could calculate with regard to each individual in turn.

(c) A third explanation can be given if we pursue the hint that the universal quantifier means " for *any* *x* ", *i.e.*, any you care to mention; we can understand any quantified proposition as a statement made in conversation, in a dialogue. Thus if Peter says to Paul that $(x)\ \phi x$, he is saying, " Pick any individual you (Paul) like, ϕ will hold for it ", whereas if he says that $(\exists x)\phi x$ he is saying, " I (Peter) can produce an individual for

which ϕ holds ". Extending this interpretation to an argument, we can take it that it is, say, Peter who asserts the conclusion, and Paul who has asserted, *i.e.* has at least conceded, the premises. Then in the example given above Peter, in asserting the conclusion, would be saying, "$A x \supset C x$ for any x that you, Paul, care to pick ". If Paul then picks, say, a, Peter has only to prove that $A a \supset C a$. Now Paul, in asserting the premises, has said that $A x \supset B x$ for any x that Peter cares to pick, and likewise that $B x \supset C x$ for any x that Peter cares to pick. So Peter now picks a in each case, and thus extracts from Paul the admissions that $A a \supset B a$ and that $B a \supset C a$, from which he proves, by the propositional calculus, that $A a \supset C a$. That is, Peter supports the conclusion by getting Paul to take up, with regard to some concrete individual, the general offer contained in that conclusion, and by then meeting this concrete demand.

However, this would have been invalid if Peter's proof that $A a \supset C a$ had relied on some distinctive feature of a, that is, if Paul had made the mistake of picking an easy example and Peter had taken advantage of this mistake. So on this third explanation, too, we can see that the argument is valid only by seeing that everything used in the proof for a was based on what Paul conceded in the premises about *any* individual.

II. The meaning of " existential instantiation ".

Of the four processes of Natural Deduction, universal instantiation (the commonsense variety) and existential generalisation are perfectly straightforward. For if ϕx holds for all x, then if a is a concrete individual ϕa must be true, and equally if a is a concrete individual and ϕa is true, then ϕx must hold for some x. The other two processes, universal generalisation and existential instantiation, are both a bit odd, and we need restrictions on the use of each of them. But the oddities are not the same, nor are the restrictions.

An argument of Rescher's helps to bring this out. As he says, if we took literally the talk about y being an " arbitrarily selected individual " we should say that if we can infer $(x)\ \phi x$ from ϕy, we can equally infer $\sim \phi y$ from $\sim (x) \phi x$, *i.e.*, from $(\exists x) \sim \phi x$. Now if we replace $\sim \phi$ with ϕ, this would enable us to infer ϕy from $(\exists x) \phi x$, where y is an arbitrarily selected individual, that is, we should have derived from U.G. a new valid argument form, existential instantiation to an arbitrary individual. But of course this is not valid, and it is not the way in

which we actually use E.I. We instantiate existentially to a real individual, not to an "arbitrarily selected" one, and that is why we can subsequently generalise only existentially.

In fact, when we argue from $(\exists x)\ \phi x$ to ϕa, we are saying: "There is at least one x for which ϕx holds; so take this thing —or any one of them—and call it a". It is as if we said, "Call it Arthur", and then went on to argue about Arthur. But we are free to use the name "a" thus, giving it to an individual about which we know nothing except that ϕ is true of it, only if the name has not previously been used in this particular context. If the name had been used before, the giving of it to the thing of which ϕ is true would not have been an arbitrary piece of name-giving, it would have been the material assertion that the thing already named a is one of those of which ϕ is true, an assertion which we have no right to make.

Since E.I. is just name-giving, there is no mystery about the use of E.I. followed by E.G., as in the following argument:

1.	$(x)\ [Ax \supset Bx]$	
2.	$(\exists x)Ax$	$\therefore (\exists x)Bx$
3.	Aa	2, E.I.
4.	$Aa \supset Ba$	1, U.I.
5.	Ba	3, 4, by *modus ponens*
6.	$(\exists x)Bx$	5, E.G.

When we thus tuck an existential quantifier up our sleeve, we go on to speak about a real individual, the proof about this individual is not merely sketched but is actually carried out, and so we can existentially generalise the result. The rule that you can apply the propositional calculus within the scope of an existential quantifier needs no further justification than this. The instantiation of premise 1 in step 4 is commonsense U.I., not U.I. (arb).

We can, however, give an explanation parallel to that in I (c) above, in terms of the dialogue interpretation of quantifiers. In the present case, Peter, in stating the conclusion, is saying, "I can produce something that is B". Now Paul, in asserting the premises, has said, "$Ax \supset Bx$ for any x that you, Peter, care to pick, and I (Paul) can produce something that is A". Peter then gets Paul to produce this something, and gives it the name "a", so that Aa is true. Next he picks this thing a as the x for which Paul has said that $Ax \supset Bx$, so that Paul has to allow that $Aa \supset Ba$. Peter infers Ba by the propositional calculus, and triumphantly produces a as the something that is B.

III. Further Problems.

The accounts already given explain some of the rules that govern generalisation and instantiation, but there are still some other restrictions—and some other liberties—to be explained. It is clear why we cannot apply U.G. to a symbol introduced by E.I., since what E.I. introduces is a genuine individual and not an "arbitrarily selected" one. But it might be supposed that there would be a parallel objection to applying E.G. to a symbol introduced by U.I.—or by U.I. (arb)—as in the following examples:

1.	$(x)\phi x$	$\therefore (\exists x)\phi x$	1.	$(x)\phi x$	$\therefore (\exists x)\phi x$
2.	ϕa	1, U.I.	2.	ϕy	1, U.I. (arb)
3.	$(\exists x)\phi x$	2, E. G.	3.	$(\exists x)\phi x$	2, E.G.

For here we are extracting from our sleeve a different quantifier from the one we tucked up it. And yet these arguments count as valid. Now it is true that we could not reach an existential conclusion without *any* existential data, but the justification lies in the assumption that there is at least one individual. When ϕa was asserted at step 2, we were implicitly both assuming that something exists and calling it a, and *then* applying 1 to it by U.I. And if U.I. (arb) is used instead, the resulting ϕy is taken, through the same assumption, to stand for a non-empty set of assertions about individuals, from any one of which step 3 would follow.

Another difficulty arises when we come to multiple quantification. Consider the two rather similar arguments:

	A.			B.	
1.	$(\exists y)(x)Rxy$	$\therefore (x)(\exists y)Rxy$	1.	$(x)(\exists y)Rxy$	$\therefore (\exists y)(x)Rxy$
2.	$(x)Rxw$	1, E.I.	2.	$(\exists y)R\,z\,y$	1, U.I. (arb)
3.	Rzw	2, U.I. (arb)	3.	Rzw	2, E.I.
4.	$(\exists y)Rz\,y$	3, E.G.	4.	$(x)Rxw$	3, U.G.
5.	$(x)(\exists y)Rxy$	4, U.G.	5.	$(\exists y)(x)Rxy$	4, E.G.

Of these, B is invalid, for it would enable us to argue from "Everything resembles something" to "There is something which everything resembles." We may at first be inclined to say that what we have done wrong is to shuffle the quantifiers while we had them up our sleeve; we should have brought out first the one we tucked up last. But then we look at argument A;

we have shuffled the quantifiers there too, and yet the argument is valid. We *can* argue from " There is something which everything resembles " to " Everything resembles something ". The problem is to explain *why* A is valid whereas B is not.

(a) Professor Copi (*Symbolic Logic*, p. 104) gives the rule that U.G. cannot be used to go from $\phi\mu$ to $(\upsilon)\phi\upsilon$ when the variable μ has a free occurrence in any earlier propositional function which was inferred by E.I., whether μ was itself introduced by E.I. or not. So far, however, this is just an *ad hoc* rule, laid down to prevent such inferences as B while leaving us free to make such inferences as A. It is still in need of justification.

(b) This can be given along the lines of the explanation in I (b) above.

At step 3 in A, since ζ is an " arbitrarily selected individual ", Rζw stands for the infinite set of propositions R*aw*, R*bw*, R*cw*, *etc.*, (where *a*, *b*, *c*, *etc.* are the various real individuals); from these, by E. G. of each in turn, we can derive the set $(\exists y)$R*ay*, $(\exists y)$R*by*, $(\exists y)$R*cy*, *etc.*, and it is for this set that $(\exists y)$Rζy in 4 stands; we need only to sum up this set by U.G. to obtain $(x)(\exists y)$Rxy in I in step 5.

But what does Rζw in step 3 of B represent? $(\exists y)$Rζy in step 2 stands for the infinite set $(\exists y)$R*ay*, $(\exists y)$R*by*, $(\exists y)$R*cy*, *etc.* Applying E.I. to the first of these we get, say, R*aw*. But if we *then* applied E.I. to the second to get R*bw*, we should be violating the already-explained, and obvious, restriction on E.I. We should be giving the same name to the occupants of two posts which we do not know to be occupied by the same individual. We should be saying, perhaps, " Call one of the things that *a* resembles *w*, and call one of the things that *b* resembles *w* too ", when we have not been told that there is anything resembled both by *a* and by *b*. Thus if Rζw in step 3 stands for the set R*aw*, R*bw*, R*cw*, *etc.*, it has been invalidly derived from step 2. And yet it is precisely this set that 3 must represent if it is to be correctly summed up by U.G. in step 4 as (x)Rxw.

If, on the other hand, step 3 is validly derived, it must represent not the set R*aw*, R*bw*, R*cw*, *etc.*, but the set, say, R*aw*, R*bv*, R*cu*, *etc.*, where each use of E.I. introduces a new name. But *this* set could not be properly summed up by U.G. as in step 4.

Thus there are two possible interpretations of Rζw in step 3 of argument B; on the first interpretation step 3 is invalidly derived from 2, though 4 would be validly derived from 3, whereas on the second interpretation step 3 is validly derived,

but 4 is invalidly derived from 3. On either interpretation the argument as a whole contains one invalid step.

We can now understand that Copi's rule presupposes that the free occurrence of z in Rzw, where the latter was introduced by E.I., indicates the set Raw, Rbw, Rcu, *etc.*, and hence that the restriction forbidding us to apply U.G. to this z is not just an *ad hoc* rule but a direct result of the meaning of U.G. There is no suitable set of results to be collected.[1]

(c) The dialogue account of quantifiers is better suited to the explaining of validity than of invalidity. It can be applied to argument A as follows, with Peter asserting the conclusion and Paul the premise as before:

Peter: For any x that you, Paul, care to pick I can produce a y such that Rxy.

Paul: I can produce a y such that, for any x you, Peter, care to pick, Rxy.

Peter: Produce your y; oh, there it is, we'll call it w. You've conceded that, for any x *I* pick, Rxw. Now you pick *your* x.

Paul: I pick a; you have to produce a y such that Ray.

Peter: First I pick a as the x for which you conceded Rxw; so you must concede Raw. And now I produce w as the y such that Ray.

As in I (c) we find here that the issue is progressively individualised, with the two speakers picking and producing in turn, until what Peter has to prove turns into the same concrete individual relation that Paul has been compelled to concede. No similar dialogue can be given for argument B, because in it Peter would have to pick his x first, and this would leave Paul free to pick a different x when his turn came. In fact the discussion would go thus:

Peter: I can produce a y such that, for any x you pick, Rxy.

Paul: For any x you, Peter, pick, I can produce a y such that Rxy.

Peter: I pick a; produce your y; oh, there it is, we'll call it w. You've conceded that Raw. So I produce w as the y for which, for any x you care to pick, Rxy. That is, pick any x you like and Rxw will be true.

Paul: I pick b; can you show that Rbw?

And of course Peter cannot.

[1] Professor Quine's alphabetic convention (*Methods of Logic*, pp. 160–161) achieves the same result, perhaps more elegantly. The two invalid proofs which he gives on p. 161 correspond to the two interpretations I have given.

Copi (*Symbolic Logic*, p. 106) states one further restriction on the use of U.G., that when $(\upsilon)\phi\upsilon$ is inferred from $\phi\mu$, the variable μ must not be free in any assumption within whose scope $\phi\mu$ lies. This too he justifies merely by pointing to the errors that would result from its violation. We can understand it, however, by considering this example, in which we appear to establish, without any premises, that if ϕ is true of the concrete individual a it is true of everything:

$\vdash\!\!\rightarrow$1. ϕy	
2. $(x)\phi x$	1, U.G.
3. $\phi y \supset (x)\phi x$	1–2, C.P.
4. $(y)[\phi y \supset (x)\phi x]$	3, U.G.
5. $\phi a \supset (x)\phi x$	4, U.I.

As in the last problem, we find that the trouble lies in a possible ambiguity. The assumption of ϕy, y being an "arbitrarily selected individual", might represent *either* (i) a set of assumptions, "Suppose that ϕa", "Suppose that ϕb," "Suppose that ϕc", *etc.*, *or* (ii) the single assumption of a set of propositions, "Suppose that ϕa and ϕb and ϕc *etc*". Now if $\vdash\!\!\rightarrow$1. ϕy has meaning (i), then 2 is invalidly derived, because it would represent the obviously invalid set of inferences:

$\vdash\!\!\rightarrow$1. ϕa	$\vdash\!\!\rightarrow$1. ϕb	$\vdash\!\!\rightarrow$1. ϕc
2. $(x)\phi x$	2. $(x)\phi x$	2. $(x)\phi x$ *etc.*

But this is precisely what is required to allow step 3 to stand for the set of propositions $\phi a \supset (x)\phi x$, $\phi b \supset (x)\phi x$, $\phi c \supset (x)\phi x$, *etc.*, from which 4 and 5 would follow correctly. If on the other hand $\vdash\!\!\rightarrow$1. ϕy has meaning (ii), then 2 is validly derived, but then step 3, if it is to be validly derived, must represent not the set of propositions above but the single proposition $(\phi a.\ \phi b.\ \phi c\ \ldots\ ) \supset (x)\phi x$, and then U.G. would yield not 4 but the harmless tautology $(x)\phi x \supset (x)\phi x$.

Thus on one interpretation step 2 is invalidly derived from 1, though 3 and 4 are in themselves correct, while on the other interpretation step 2 is correct, but either 3 or 4 (according to the meaning assigned to 3) is incorrect. On either interpretation the argument contains one invalid step. Copi's rule can be understood as the acceptance of interpretation (i) and its consequences.

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SEA-FIGHTS WITHOUT TEARS

By JOHN KING-FARLOW

ARISTOTLE was concerned to safeguard human freedom no less in expounding logic than in exploring ethics. There have been many discussions of his paradoxical thesis at *De Interpretatione*, Chapter Nine that certain assertions about future events can be neither true nor false yet the disjunction of such assertions and their denials must be necessarily true. Further investigation may still prove fruitful: it is here suggested that underlying the contestable meaning of the Greek text we can find a neglected series of logical relations between different types of ' propositions '. And that such a series, highly relevant to perennial topics like Timeless Truths and Fatalism, might constitute a vindication of something rather similar to the original paradox.

The best way to uncover these relations is first to consider certain efforts of logicians to obscure the categorial distinctions by trying to resurrect Aristotle in a multi-valued logic. At the close of his thorough and sympathetic exposition of Lukasiewicz's three-valued attempt Professor A. N. Prior concluded that the Polish notation can only serve to bring out the first but not the second of Aristotle's theses. For " as we have already noted, ApNp is not one of the laws of the Lukasiewicz-Tarski 3-valued system—ApNp=1 when P=0 or 1 but, when $P=\frac{1}{2}$, $ApNp=A\frac{1}{2}N\frac{1}{2}=A\frac{1}{2}\frac{1}{2}=\frac{1}{2}$ " (*Philosophical Quarterly*, 1953, p. 326). A good deal earlier Professor C. A. Baylis argued that Lukasiewicz's notation was not needed anyway to ensure the non-necessary character of true propositions about the future—two truth-values were quite adequate once we have distinguished being ' true ' and being ' knowable ' (*Philosophy of Science*, 1936, p. 162). But, I shall argue, a ' Category Mistake ' is involved in holding that either the Polish symbols or those of *Principia* can reflect the real logical truths of which Aristotle's theses are themselves very imperfect images.

Perhaps a parable will illuminate the desire to ' save ' future contingency with a third value. Once upon a time Citizen Kane built himself a private zoo, dominated by an extensive ape-house. When Lord Raspborough sent him an enormous gorilla (then thought to be the Abominable Snowman) Kane erected an adjoining annex to the ape-house twice the height and size of

the original. It seemed only logical that all the simian specimens be housed under what was technically the same superstructure. But when a visiting Maharajah presented him with an unknown creature of immense proportions we might expect Kane in the press of graver affairs to do the irregular thing and house the new beast in the Ape House Extension as the only place large enough to contain it. So too logicians have enlarged their truth tables to admit legitimate specimens of what Waismann calls Truth Distance (" yellow is not quite the right colour "), which are neither true nor false; so too they are then tempted to accommodate in their new structure expressions which are neither true nor false nor strictly propositions at all. This is what I mean by the tedious phrase ' Category Mistake '. What has this to do with ' there will be a sea-fight tomorrow '? Let us break the spell of our language by constructing another.

Suppose yours is a tenseless foreign language and you say ' John is married to Mary in 1959 ' both in 1958 when they're single, then in 1959 when they're married. Suppose in your national symbolic language you represent the latter (in a less abbreviated form) as P^{59} and the former as P^{59} or $/P^{59}/$ depending on the context.[1] Suppose too that in the case of certain natural phenomena like the sun rising, stones falling and fire burning you represent these in 1958 always as R^{59}, F^{59}, B^{59}, never in any context as $/R^{59}/$, $/F^{59}/$, $B^{59}/$. This is because you think of these as necessary by decree of nature. " Well," I say, " when the marriage comes off in 1959 I guess you say that P^{59} or $/P^{59}/$ was true." " Oh, no," you reply, " we do not say ' $/P^{59}/$ was true ' but ' $/P^{59}/$ came true '; just so our language never says that $*Q^{59}*$ (meaning something like ' I assure you I'll repay in 1959 ') was true but that $*Q^{59}*$ was made good. Now for us to say $/P^{59}/$ and subsequently they don't marry or $*Q^{59}*$ and subsequently I go bankrupt is very bad and people who make such talk too often are beaten with rods. So if Mary is a fickle sort of girl I only say $//P^{59}//$ or even $////P////$, then people will not be angry if eventually I'm proved wrong. If I see a business recession on the horizon I say $***Q^{59}****$ which means something like ' Well, I'll do my best to repay you in 1959 but I make no promises, so please be patient.' You see with our people degrees of assurance are very important in the national code of honour. So compare our signs like $\%P^{59}\%$, (' there's a faint chance they'll marry '), $\$P^{59}\$$ (' there's an interesting possibility they'll

[1] In much of what follows I am indebted for criticisms to Messrs. R. L. Clark, John Heintz and Richard Severens of Duke University.

marry '), and so on. And all these funny signs are represented in our spoken language by tones of voice or accompanying gestures or little particles that get prefixed and suffixed to the tenseless verbs.

" But when do we say P^{59} in '58? Well, I assert 'if A^{59} and Z^{59} then P^{59} ' quite naturally today; for that means: ' if they go to the marriage maker and he performs the ceremony in 1959 then they'll be married in 1959.' But that's an even safer bet than the sunrise: that's something I can know now for sure by mere definition. So too I can say P^{59}-or-not-P^{59} and to qualify this as /P^{59}-v-not-P^{59}/, (like your ' I hazard a guess they will or won't '), would seem, if said seriously, what you call neurotic. To say /P^{59}/v/not-P/^{59} would be still queerer, something like your ' either I guess that P or I guess that not-P '. Again when I wish to indicate that there's something rather uncannily inevitable about the marriage I say ' P^{59} as sure as R^{59} '; or if I am stating an intention by announcing a meeting or I am describing to tourists a ceremony that will take place at the Coronation next year I would say E^{59}, X^{59}, G^{59} without qualifiers—but here I sometimes feel I am talking about the necessary consequent of an odd sort of suppressed hypothetical in an odd sort of semi-tautology—' if all goes to plan then. . .'? But very often in our language we qualify these so: ()G^{59}()—which is hard to translate in your language but means roughly: this isn't a real future statement but an expression of our intention to hold a meeting in 1959.

" There are all sorts of odd cases where you might say P^{59} today and so be making the sort of statement about the future we call true or false. Fortune telling, speaking with extra authority as parents to our children or as officers or experts to the broad masses. But people who do this often are considered pretentious, pompous and even hypocritical. Of course the unqualified future is the *primitive* form of our language and indispensable in that without learning it we could not come to understand the qualified ones. But when we talk about the future we often want to make statements about events which involve an element of uncertainty, especially those where we feel people are free to choose and, if these are not really statements of intention or like consequents of suppressed quasi-tautologies and so on, we make this clear with the qualifiers. And, to let you in on what is almost a secret, our sacerdotal hierarchy encourages the use of these qualifiers as a remedy to certain fatalist heresies that sometimes get a grip on our people."

There are four points of especial relevance about this language of yours. First, it embodies a metaphysic in an important way analogous to Aristotle's own contrast of natural laws with events dependent on human choice as first cause, *ἀρχή*. Second, the operation of the qualifiers is illuminatingly similar to that of Professor Austin's performatives or the quasi-performative functions Professor Toulmin has shown to belong to ' probably'. Their most striking effect is to withold the apparent ' proposition ' from the calculus of truth and falsehood.

Third, many people make statements about the future which, they would agree on reflection, are virtually qualified by an Austinian performative, an Urmsonian parenthetical operator or a ' probably ' in Toulmin's sense. Such people would agree with Professor Ryle in *Dilemmas* that these are the sort of statements of which it is less misleadingly said (*post eventum*) that they came true or were fulfilled, rather than that they were true, the day before. Their future is not really described or stated but (implicitly) ' guessed ', ' predicted ' or ' forecast ', to quote three of Ryle's most frequently recurring verbs. Such predictions are only ' true ' in a secondary sense. This is *their* way out of the Fatalism Paradox. Others (like Ayer when he discusses Precognition in *The Problem of Knowledge*) are happy to take the bull by the horns and say that even if we could have knowledge of the future it would not necessarily be determined: I could precognise a truth on Monday about John's free choice on Tuesday. Others like Baylis feel that a crucial error about determinism is dispelled if we only distinguish the ' true ' from the ' known ': they find comfort in his reminder that there are numerous facts about the past we do not know and our acquaintance with a fact is only necessary for our knowing a proposition to be true, not for its being true (*loc. cit.*, p. 162).

All this only serves to show the futility of talking Kant-wise about ' what is really thought in the universal concept ' of knowing or being true. For it is just because different people's concepts sometimes interlock in different ways that some do and some don't find Ayer's or Baylis' therapy helpful.[1] An Aristotelian (and perhaps a Rylean) might put it something like this: " What I am trying principally to safeguard is the concept of human choice as *ἀρχή*. Therefore I say that statements of future contingency are not true or false. Statements about the past are a false analogy because the past can't be changed and they

[1] For a longer attack on ' absolute ' interpretations of synthetic-analytic v. my discussion in *Philosophical Quarterly* 1957, pp. 137–8.

are in principle verifiable now, in principle knowable by a hypothetically omniscient temporal being whereas certain future ones are in principle not knowable, and thus not true. My concepts of the ' true ' and ' knowable ' imply that the true is knowable now for sure and therefore fixed or pegged (cf. Sartre's notion that my past is ' figé en soi '); what is fixed excludes human choice as ἀρχή. Therefore I beware of future ' truths '."

The first tenet of Aristotle, that ' there will be a sea-fight tomorrow ' is neither true nor false, can now be symbolized properly. Wittgenstein says ' we call something a proposition when *in our language* we apply the calculus of truth functions to it.' (*P.I.*, 136) Well, then, in your tenseless and contingency-conscious language let SF^{+1} represent the statement without a qualifier and let us assume you use the good old British military abbreviation N/A for ' not applicable '. You have the following table of values:

	SF^{+1}	Not-SF^{+1}	/SF^{+1}/	/Not-SF^{+1}/	*SF^{+1}*	etc., etc.
T	T	F	N/A	N/A	N/A	N/A
F	F	T	N/A	N/A	N/A	N/A

Aristotle's metaphysical point as reflected in your language is that man is by the nature of things and *in principle*—whatever his practical difficulties about the past and present—only in a position to make quasi-qualified future statements about certain classes of events. (SF^{+1} has very limited use). He cannot often legitimately formulate ' propositions ' about them. The logical corollary is reflected in these tables and would have to be so reflected in a multi-valued table—however many the values we would still register only N/A's in the /SF^{+1}/ or *SF^{+1}* column! Indeed Toulmin's quasi-performative exposition of ' probability ' which applies partly to ' possibility ', suggests that a certain amount of modal logic needs to be similarly recast to avoid categorial confusion.

Fourth, Aristotle's second thesis that the disjunction of a future statement with its contradictory is necessarily true though the statement alone is neither true nor false is fairly well illuminated by your imaginary language. A blind man is in a position to know as a *non-informative* logical truth that his apple is either entirely green or not entirely green. Any man has similar knowledge that there will or won't be a sea-fight tomorrow. And it is just this non-informative character of his knowledge that entitles him even on the Aristotelian metaphysic to make an unqualified future statement, i.e., a genuine

' proposition '. But, as we have already been led into constructing a symbolism of quasi-performative operators, it is worth suggesting that Aristotle unwittingly points to a whole network of logical truths and relations which can only be hinted at here.

In Austin's jargon a performative expression about repayment like $*Q^{59}*$ may be ' *infelicitously formed* ' for a wide number of reasons, because I intend to commit suicide in a week or because you're not the chap I owe anything to, and so on. So may $/P^{59}/$ or even $///P^{59}///$ if John is my dead brother or Mary is the old grey mare. Caligula could felicitously vow to make his horse a consul but not to make his horse a female ox. Without much difficulty we could draw up Felicity Tables with numerous degrees, values, of infelicity. We can see that the logical rules which we normally apply to unqualified ' propositions ' have much bearing on the felicity, and the inter-relations, of performatively or quasi-performatively pegged statements.

For instance, let K represent (tenselessly) ' you meet my son Herbert ' and M represent ' you meet one of my children ', and let ' son ' and ' children ' and ' my ' be so understood that $K \supset M$. (What I say about ' $\supset$ ' here applies only to its use in strict logical entailments, not to its use in ' meaty hypotheticals ' like ' if Jim's drunk he'll be rude '.) Thus if I predict $/K^{59}/$ I implicitly predict $/M^{59}/$ with at least the same degree of assurance: ' $/K^{59}/$ but $//M^{59}//$ ' is infelicitously formed. For if I say ' $K \supset M$ ' and ' $/K^{59}/$ ' then ' $/M^{59}/$ ' is guaranteed *via* (let us say, rather than by) the meaning of ' $\supset$ '. That is: $[(K \supset M) . /K^{59}/] \rightarrow /M^{59}/$'; ' $\supset /M^{59}/$ ' might obscure the importance of categorial distinctions. Similarly *via* the meaning of ' . ' in propositions the law of contradiction guarantees the infelicity of ' $/P^{59} . \text{not-}P^{59}/$ ' or $*R^{59} . \text{not-}R^{59}*$. Again *via* the usual meaning of ' v ' a ' $/P^{59} \text{v not-}P^{59}/$ ' is guaranteed the status of what we'd call a ' dead safe bet '. But whereas propositional logic makes the use of the qualifier in $/P^{59} . \text{not-}P^{59}/$ illegitimate and outrageous, it makes its use in $/P^{59} \text{ v not-}P^{59}/$ only unnecessary and somewhat silly. A dead safe bet of the latter kind is too much of a good thing —since my ' position ' (a key word in Austin's account of ' I know ') is altogether too good, the appropriate contextual background for the predictive qualifier disappears in an infelicitous surfeit of pseudo-felicity. Hence Aristotle's second thesis somewhat somnambulantly leads us to the delicate relations between propositional logic and the status of performatives and quasi-pegged statements. The metaphysical structure as reflected in your language is that we are in principle able to know

that ' it will or won't happen ' is true without compromising ἀρχή. The linguistic corollary is that such ' predictions ' are pointless. Let :: mean ' has no more point than ' and we symbolize: /P^{59}v not-P^{59}/::P^{59} v not-P^{59}.

Aristotle's metaphysical theses about the nature of things and the epistemological relation of man to different classes of events may or may not interest logicians on their day off. At any rate he was led by his metaphysic to intertwine, rather obscurely, logical theses which one can fruitfully pursue further. The work of Wittgenstein, Austin, and Toulmin among others points to the need not for linguistic snobbery, but for an approach to symbolism which discards the overweening hegemony of propositions. By reducing the proposition to a constitutional monarch we can resolve the enigma of the sea-fight and move closer to some genuine metaphysical questions.

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IMPERATIVES AND DEONTIC LOGIC[1]

By HECTOR NERI CASTANEDA

THIS is a critical follow-up of Peter Geach's discussion of imperative logic in a recent paper.[2] I want to consider:

(i) Geach's suggestion that we could dispense with imperatives in favour of assertions in the future tense (IDL, p. 51);

(ii) two rules of inference mentioned by R. M. Hare[3] (LM, p. 28; IDL, pp. 52–56).

I

Geach suggests that imperatives can be reduced to future-tense statements as one line of argument against Hare's view concerning the relationship between imperative logic and deontic logic. He claims that (G) the identity of a future-tense statement's " coming true " with the fulfilment of an imperative is *the* source of our saying that imperatives can be inferred, that

[1] I am grateful to G. Nakhnikian for his valuable criticisms of both content and style.

[2] P. T. Geach, ' Imperative and Deontic Logic ', ANALYSIS, Vol. 18 (1958), pp. 49–56 (to be referred to hereafter as IDL).

[3] R. M. Hare, *The Language of Morals* (Oxford; Clarendon Press, 1952) (to be referred to as LM).

they can be compatible, etc. For this reason, he says that " the logician is interested only in the fulfilment as such, not in the difference between intending, being commanded, and being forewarned." Thus, " the same utterance of the sentence ' the patient will go down to the operating theatre ' may be at once an expression of the surgeon's intention, an order to the nurse, and a piece of information to the patient." (IDL, p. 51).

Now it does seem plausible to say that the imperatives " Paul, read Ryle's *Concept of Mind*" and " Paul, do not read Ryle's *Concept of Mind*" are contradictory because it is logically impossible to fulfil both (at the same time). Thus, Geach's claim (G) does seem to have some foundation.

Now, one does not have to deny that the fulfilment of imperatives is *at* the source of the logical properties of imperatives in order to reject (G). Perhaps the above mentioned imperatives are contradictory because the statements " Paul has read Ryle's *Concept of Mind*" and " Paul has not read Ryle's *Concept of Mind*" are contradictory (in the primary sense of this word—they cannot both be true). But this does not mean that something other than fulfilment is not part of the source of the logical properties of imperatives, e.g., other aspects or features of commands, requests, invitations together with certain aspects of the context of utterance of imperatives formulating such commands, requests, etc. I will not discuss here what these other aspects are; but that fulfilment alone is insufficient to explain certain logical relationships among imperatives is clear from the following examples:

(1) If you read the book, come to see me.

Read the book.

Ergo, come to see me.

(2) If you (will) read (or have read) the book, you will (or have) come to see me.

You will (or have) read the book.

Ergo, you will (or have) come to see me.

It is easy to see that inference (2) is valid, and that it is immaterial whether the statements in (2) are about the past, the future or the present. On the other hand, (1) is invalid in the important sense that a teacher who tells or orders his student both to come to see him if he reads a certain book and to read that book has

not thereby ordered or told his student to come to see him, regardless of the student's reading of the book. The student will be drawing the wrong conclusion if just a few minutes later he goes to see his teacher on the grounds that the latter issued the two orders formulated in the premises of (1). Nevertheless, if (G) were correct the source of the validity (or invalidity) of (1) would lie entirely in (2). Since (2) is valid, (G) and (i) would make (1) valid too. But since (1) is invalid, I conclude that imperatives cannot be dispensed with in favour of future-tense assertions without having to make a distinction between two irreducible types of future-tense assertions: (a) predictive and (b) imperative.

In the sense in which (1) is invalid (3) below is valid.

(3) If you read the book, come to see me.

You have read the book (already).

Ergo, come to see me.

Clearly, the first premise of (3) formulates an order or invitation, the second premise is true and their relation to the conclusion is such that the latter cannot fail to formulate an order or invitation, too. The phrases ' formulates an injunction (or command, or request, or invitation, etc.) ' must be subjected to an examination, but since we have to begin somewhere, we may start from them in the present context and establish the following provisional criterion which covers the validity of imperative inferences (if we are to recognize them at all):

(C) An inference is valid only if it is impossible for both the conclusion to be false or not to formulate an injunction, or command, or request, or invitation and all the premises to be true or to formulate an injunction, or command, etc.

II

Hare has mentioned the following principles of imperative inference (LM, p. 28):

(I) No indicative conclusion can be validly drawn from a set of premises which cannot be validly drawn from the indicatives among them alone;

(II) No imperative conclusion can be validly drawn from a set of premises which does not contain at least one imperative.

Henceforth I shall refer to (II) as Poincaré's principle, for as Hare has pointed out it was Poincaré[1] who first made a clear statement of (II). (I) is an invention of Hare's.

I fully agree with Geach that (I) is false. Geach gives several examples in *modus tollens* (IDL, p. 52), e.g.,

(4) If you are a faithful subject, then rise up, Sir George!

But do not rise; stay on your knees, fellow!

Ergo, you are not a faithful subject.

I also agree with Geach that (II) is not fully correct, as a consideration of so-called hypothetical imperatives shows—in spite of Hare's protestations to the contrary. But I do not want to discuss hypothetical imperatives here. All I want to say is that Geach's discussion of (II) misses something. He offers two proofs (IDL, pp. 55–56) of the validity of

(5) Grimbly Hughes is the largest grocer in Oxford;

Ergo, either do not go to the largest grocer in Oxford or go to Grimbly Hughes.

Clearly, if (5) is valid, rule (II) does not govern imperative inferences. Geach's proofs of the validity of (5) are based on the following principles which he calls *themata*:

(A) *Thema II*: If the inferences of the form ' p, q, *ergo* r ' are valid; then, inferences of the form ' p, *ergo* either not q or r ' are also valid; and

(B) *Thema III*: If the inferences of the form ' Either not p or p, q, *ergo* r ' are valid, then inferences of the form ' q, *ergo* r ' are valid.

Doubtless, (A) and (B) govern inferences in which p, q, and r are statements. If (G) and (i) are correct, we could, as Geach does, speak of the future-test for imperatives, and surely imperatives would conform to (A) and (B). Now Geach explicitly says that he is not assuming the correctness of (i) or the future-test in his discussion of rule (II) (cf. IDL, pp. 52, 53). However, Geach nowhere establishes that (A) and (B) also apply to imperatives, regardless of whether (i) is correct or not. Thus, his proofs that (5) is a valid inference are somewhat dogmatic.

In fact, Geach is more concerned with showing that the

[1] H. Poincaré, ' La Science et la Morale ', *Dernières Pensées*.

conclusion of (5) is not futile than with showing that (A) and (B) apply to imperatives. He argues that such a conclusion is only practically vacuous or futile, that " a logician determined to show, by disobeying the ' Grimbly Hughes ' imperative, that it was not practically futile could do so by getting a friend in the I.R.A. to blow up Grimbly Hughes, and then visiting the *then* largest grocer in Oxford but not the ruins of Grimbly Hughes ". (IDL, p. 55; his italics). I fail to see the force of this argument. When Geach's logician visits the largest grocer in Oxford, the premise of (5) is false, so that even if (5) is valid, we cannot say very much about its conclusion. All that Geach's case establishes is that both the premise and the conclusion of (5) have a temporal reference; so let us suppose that it is made explicit by the insertion of the word ' now ' or some other temporal expression at the proper places.

I take it that Geach's concern with the futility of a command, request, etc., is analogous to our ordinary interest in statements which are synthetic, i.e., not logically true or self-contradictory. Indeed, the point of Poincaré's principle seems to be that from statements of fact alone we cannot derive genuine commands or requests, where ' genuine ' is used to express that the agent is presented with a definite course of action, i.e., he can choose to obey or choose to disobey such commands, etc. Clearly, the imperative

(6) Do not go to the largest grocer in Oxford or go to Grimbly Hughes

may be said to be synthetic or to provide genuine advice or command if Geach's claim (G) is correct—for certainly the indicative ' You do not go to the largest grocer in Oxford or you go to Grimbly Hughes ' is synthetic or contingent. Also, if we accept (G), (5) turns out to be valid.

Let us suppose that the conclusion of (5) is not vacuous. Let us suppose that (6) ' Either do not go to the largest grocer in Oxford, or go to Grimbly Hughes ', formulates a genuine command.

Now, if (5) is valid without any restriction, its validity is independent of the proper name you put as the subject of sentence (6). Consider, e.g., the following:

(5a) Grimbly Hughes is now the largest grocer in Oxford; *ergo*, Mao Tse-tung, either do not go now to the largest grocer in Oxford or go now to Grimbly Hughes.

Since it is true that Grimbly Hughes is the largest grocer in Oxford, the conclusion of (5a) holds, i.e., it formulates a genuine command or injunction to Mao Tse-tung which, on Geach's claim, he could in principle fail to obey. Similarly, all Patagonians are under the command to go to Grimbly Hughes or not to go to the largest grocer in Oxford. But this is most incredible if as Geach claims, (6) is not a vacuous command. For that matter, it is not even clear why the undergraduates at Wadham College are all commanded to go to Grimbly Hughes or not to go to the largest grocer in Oxford, just because it happens that Grimbly Hughes is the largest grocer in Oxford. Thus, if (6) formulates genuine advice or command, which cannot apply to thousands of people, then (5) does not satisfy (C) and is, therefore, an invalid inference. Hence, principles (A) and (B) above cannot govern imperative inferences without some qualifications or restrictions. Surely, the formulation of such restrictions would be the formulation of special principles of imperative logic.

Why is it odd to say that a young boy in Bagdad who has never heard of England is under the command (or request or invitation or entreaty) either not to go to the largest grocer in Oxford or to go to Grimbly Hughes? The reasons for this oddness are essentially that (*a*) an imperative sentence like (6) must be used to perform one or another of the " prescriptive " acts: commanding, requesting, inviting, entreating, advising, etc.; and (*b*) there must be a person or institution who issues the command or makes the request or gives the advice, etc. Facts or statements of facts by themselves do not seem to entail commands, requests, advice in the absence of a person, a legislator or other authority who uses imperative language. That is the point of Poincaré's principle (II), and I think that it is a good point. To be sure, we can, for the sake of symmetry with descriptive logic, allow a harmless exception to (C) and (*b*) and say that logically futile or analytic imperatives like ' Either go to Grimbly Hughes or do not go to Grimbly Hughes ' do follow from statements of fact.

On the other hand, if (6) is vacuous the validity of (5) contrasts sharply with the validity of (3), whose imperatives do formulate genuine commands, invitations, etc. Moreover, it is important to notice that the vacuousness of (6) and of the conclusions of (5) and (5a) does not make the statement ' Either Eisenhower does not go to the largest grocer in Oxford or he goes to Grimbly Hughes' vacuous. This statement is synthetic

and makes a definite, genuine report. Whereas if (6) is vacuous, then (5) and (5a) are valid, but then a synthetic imperative in accordance with (G) and (i) may fail to formulate a genuine command, request, etc. Clearly, this is similar to the point contained in the so-called Kantian principle that 'ought' implies 'can'. The 'can' implied by 'ought' is not just the logical 'can', but a 'can' of physical or nomological possibility. That is to say, Kant's principle is taken to assert that for an 'ought'-assertion like 'X ought to do A' to formulate a duty it does not suffice that 'X does A' is a synthetic proposition; *inter alia*, 'X does A' must be consistent with propositions describing X's abilities.

Thus, it seems that the logic of non-vacuous imperatives, like the logic of non-vacuous deontic assertions, is somewhat different from the logic of non-vacuous descriptive or factual indicatives. At any rate, if (6) is vacuous because it is impossible to disobey it, then we have here only a trivial exception to Poincaré's principle—even though Hare's formulation of it is certainly a bit too strong.

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